## MATRICES

MATRIX: A matrix is an ordered rectangular array(i.e. arrangements of numbers or functions)such as

$$
\mathrm{A}=\left[\begin{array}{ccc}
2 & \cos x & \sqrt{3} \\
1.1 & 4 & 0
\end{array}\right]
$$

Matrices are represented by capital letter A,B,C..etc.
The numbers or functions in a matrix are called elements of the matrix. The elements in the above example are $2, \cos x, \sqrt{3}, 1.1,4$ and 0 .

A horizontal line of elements is called row of the matrix and the vertical line is called column of the matrix.
The matrix in the above example has two rows and three columns.
ORDER OF THE MATRIX: A matrix having $m$ rows and $n$ columns is called a matrix of order $m \times n$.

The matrix in the above example has order $2 \times 3$.
Order of the matrix $C=\left[\begin{array}{cc}1 & 2 \\ -1 & 2 \\ 0 & 5\end{array}\right]$ is $3 \times 2$.
Order of the matrix $A=\left[\begin{array}{l}5 \\ 4 \\ 3\end{array}\right]$ is $\qquad$
In general , a $\mathrm{m} \times \mathrm{n}$ matrix is written as $\mathrm{A}=\left[\begin{array}{ccc}a_{11} & \cdots & a_{1 n} \\ \vdots & \ddots & \vdots \\ a_{m 1} & \cdots & a_{m n}\end{array}\right]$
Matrix can be written in compact form as $A=\left[a_{i j}\right]_{m} x_{n}$. the number of elements in $m \times n$ matrix is $m n$.
The element $\mathrm{a}_{\mathrm{ij}}$ is in the $\mathrm{i}^{\text {th }}$ row and $\mathrm{j}^{\text {th }}$ column and it is called $(1, \mathrm{j})^{\text {th }}$ element.

## TYPES OF MATRICES:

1. ROW MATRIX: A matrix having one row only.

Example $A=\left[\begin{array}{lll}-7 & \sqrt{2} & 1\end{array}\right]$
2. COLUMN MATRIX: A matrix having one column only.

Example: $\mathrm{B}=\left[\begin{array}{l}5 \\ 4 \\ 3\end{array}\right]$
3. SQUARE MATRIX: A matrix is said to be square if it has same number of rows and columns.

For example: $A=\left[\begin{array}{cc}1 & 5 \\ -9 & 2\end{array}\right]$ or $B=\left[\begin{array}{ccc}1 & 1 & 3 \\ 8 & 0 & -5 \\ 7 & 8 & 99\end{array}\right]$
The elements $a_{11}, a_{22}, a_{33} \ldots .$. . are called diagonal elements . The line along which the diagonal elements lie is called principal diagonal.
4. DIAGONAL MATRIX: it is a square matrix in which all diagonal elements are non zero and rest all zeros.

Ex: $C=\left[\begin{array}{lll}1 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 9\end{array}\right]=\operatorname{diag}(1,6,9)$
5. SCALAR MATRIX: A square matrix is called scalar if diagonals elements are same and rest all zeros.

For ex: $X=\left[\begin{array}{lll}4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4\end{array}\right]$
In other words, a scalar matrix is a diagonal matrix with identical diagonal elements.
6. IDENTITY MATRIX (Unit Matrix): A square matrix is called identity matrix or unit matrix if all the non diagonal elements are zero and diagonals elements are unity.

The identity matrix of order n is denoted by $\mathrm{I}_{\mathrm{n}}$.
For ex: $I_{2}=\left[\begin{array}{ll}1 & 0 \\ 0 & 1\end{array}\right]$ and $\mathrm{I}_{3}=\left[\begin{array}{lll}1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1\end{array}\right]$ are identity matrices of order 2 and 3.
7. NULL/ZERO MATRIX: A matrix whose all elements are zero is called null matrix or zero matrix it is denoted by 0 . For ex : $\mathrm{O}=\left[\begin{array}{lll}0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0\end{array}\right]$. It can be of any order.
8. UPPER TRIANGULAR MATRIX: A square matrix $A$ is said to be an upper triangular matrix if all the elements lying below the main diagonal are zero. ie. If $\mathrm{a}_{\mathrm{ij}}=0$ for $\mathrm{all} \mathrm{i}>\mathrm{j}$.
for ex: $\mathrm{A}=\left[\begin{array}{lll}1 & 5 & 5 \\ 0 & 4 & 6 \\ 0 & 0 & 9\end{array}\right]$.
9. LOWER TRIANGULAR MATRIX: A square matrix $A$ is said to be an lower triangular matrix if all the elements lying above the principal diagonal are zero. ie. If $a_{i j}=0$ for all $i<j$.
for ex: $A=\left[\begin{array}{lll}1 & 0 & 0 \\ 3 & 4 & 0 \\ 7 & 8 & 9\end{array}\right]$

## ASSIGNMENT-1

1. let $A=\left[\begin{array}{ccc}3 & -7 & -3 \\ \sqrt{3} & 55 & 2.5 \\ 1 & 0 & 6\end{array}\right]$ be any matrix

Write i) order of matrix $\quad$ ii) number of elements iii) elements $a_{12}, a_{32}, a_{21}$.
2. If a matrix has $\mathbf{1 8}$ elements, what are the possible orders it can have? What if it has $\mathbf{1 1}$ elements?
3. construct a $3 \times 4$ matrix $A=\left[a_{i j}\right]$, whose elements are given by $a_{i j}=\frac{|i-j|}{2}$
4. Write the element $a_{12}, a_{22}$ of matrix $A=\left[a_{i j}\right]$ whose elements $a_{i j}$ are given by $a_{i j}=e^{2 i x} \operatorname{sinjx}$.
5. construct a $3 \times 4$ matrix whose elements are given by $\mathrm{a}_{\mathrm{ij}}= \begin{cases}i+j & \text { if } i \geq j \\ \frac{1}{2(i-j)} & \text { if } i<j\end{cases}$
6. Write the number of all possible matrices of order $2 \times 3$ with each entry 1 or 2 .
7. What is the number of all possible matrices of order $3 \times 3$ with each entry 9 or 8 ?
8. write the number of all possible matrices of order $2 \times 2$ with each entry 2,3, or 4 .
9. Give an example of : i) scalar matrix ii) diagonal matrix iii) row matrix which is also a column matrix
10. If $A=\left[\begin{array}{ccc}3 & 2 & 0 \\ -1 & 1.5 & 4 \\ 7 & 1 & 2\end{array}\right]$ and $B=\left[\begin{array}{ccc}0 & 22 & 8 \\ 7 & 2 & 5 \\ 8 & 0 & 2\end{array}\right]$, then find $a_{22}+b_{21}$.

## EQUALITY OF MATRICES:

Two matrices $A$ and $B$ are said to be equal if their order is same and corresponding elements are equal i.e $a_{i j}=b_{i j}$. If $A$ and $B$ are equal matrices, then we write $A=B$. Otherwise we write $A \neq B$

## ASSIGNMENT-2

1. Find the values of $x, y$ and $z$ if $\left[\begin{array}{ll}4 & 3 \\ x & 5\end{array}\right]=\left[\begin{array}{ll}y & z \\ 1 & 5\end{array}\right]$
2. Find the value of $x$ if $\left[\begin{array}{cc}3 x+y & -y \\ 2 y-x & 3\end{array}\right]=\left[\begin{array}{cc}1 & 2 \\ -5 & 3\end{array}\right]$
3. Write the value of $x-y+z$ from the equation: $\left[\begin{array}{c}x+y+z \\ x+z \\ y+z\end{array}\right]=\left[\begin{array}{l}9 \\ 5 \\ 7\end{array}\right]$
4. Find the values of $a$ and $b$ if $A=B$, where $A=\left[\begin{array}{cc}a+4 & 3 b \\ 8 & -6\end{array}\right]$ and $B=\left[\begin{array}{cc}2 a+2 & b^{2}+2 \\ 8 & b^{2}-5 b\end{array}\right]$
5. If $A=\left[\begin{array}{cc}\cos \emptyset & -\sin \varnothing \\ \sin \emptyset & \cos \emptyset\end{array}\right]$, then for what value of $\emptyset$ is $A$ an identity matrix?
6. Find the values of $\mathrm{a}, \mathrm{b}, \mathrm{c}$ and d from the equation: $\left[\begin{array}{cc}2 a+b & a-2 b \\ 5 c-d & 4 c+3 d\end{array}\right]=\left[\begin{array}{cc}4 & -3 \\ 11 & 24\end{array}\right]$

ADDITION OF TWO MATRICES: Let $A=\left[a_{i j}\right]$ and $B=\left[b_{i j}\right]$ be two matrices of same order, then we define sum of matrices $A$ and $B$ as $A+B=\left[a_{i j}+b_{i j}\right]$.

In simple words, $A+B$ is a matrix obtained by adding the corresponding elements of $A$ and $B$.
Let $A=\left[\begin{array}{cc}1 & 2 \\ -1 & 2\end{array}\right], B=\left[\begin{array}{cc}4 & 1 \\ 0 & -2\end{array}\right], C=\left[\begin{array}{ccc}2 & 3 & 5 \\ 1 & 0 & -2\end{array}\right]$ then $\mathrm{A}+\mathrm{B}=\left[\begin{array}{cc}5 & 3 \\ -1 & 0\end{array}\right]$ but
$A+C$ and $B+C$ are not defined as their order is not same.
NOTE: i) If $A$ and $B$ are not of same order, then $A+B$ is not defined.
ii) Matrix addition is commutative: $\mathrm{A}+\mathrm{B}=\mathrm{B}+\mathrm{A}$
iii) Matrix addition is associative: $A+(B+C)=(A+B)+C$

SUBTRACTION OF TWO MATRICES: Let $A=\left[a_{i j}\right]$ and $B=\left[b_{i j}\right]$ be two matrices of same order, then we define difference of matrices $A$ nd $B$ as $A-B=\left[a_{i j}-b_{i j}\right]$.

In simple words, $A-B$ is a matrix obtained by subtracting the corresponding elements of $A$ and $B$.
NOTE: If $A$ and $B$ are not of same order, then $A-B$ is not defined.
Let $A=\left[\begin{array}{cc}1 & 2 \\ -1 & 2\end{array}\right], B=\left[\begin{array}{cc}4 & 1 \\ 0 & -2\end{array}\right], C=\left[\begin{array}{ccc}2 & 3 & 5 \\ 1 & 0 & -2\end{array}\right]$ then $A-B=\left[\begin{array}{cc}-3 & 1 \\ -1 & 4\end{array}\right]$
but $A-C$ and $B-C$ are not defined as their order is not same.
SCALAR MULTIPLICATION (multiplication of a matrix by a scalar): Let $\mathrm{A}=\left[\mathrm{a}_{\mathrm{ij}}\right]$ e a matrix of any order and let k be ay scalar, then we define multiplication of matrix $A$ by scalar $k$ as $k A=\left[k a_{i j}\right]$

In simple words, kA is a matrix obtained by multiplying each element of A by scalar k .
For ex: let $A=\left[\begin{array}{cc}1 & 2 \\ -1 & 2\end{array}\right]$. Then $2 A=2\left[\begin{array}{cc}1 & 2 \\ -1 & 2\end{array}\right]=\left[\begin{array}{cc}2 & 4 \\ -2 & 4\end{array}\right]$.

## ASSIGNMENT-3

1. If $A=\left[\begin{array}{cc}-1 & 0 \\ 3 & 9\end{array}\right]$ and $B=\left[\begin{array}{cc}2 & -5 \\ -3 & 5\end{array}\right]$, then find $A+B, A-B, 3 A$
2. If $A=\left[\begin{array}{cc}-1 & 0 \\ 4 & 5\end{array}\right]$ and $B=\left[\begin{array}{cc}0 & -5 \\ -3 & 7\end{array}\right]$, then find $3 A-2 B$
3. If $\mathrm{A}=\operatorname{diag}(-2,5,1)$ and $\mathrm{B}=\operatorname{diag}(1,0,-4)$, then find $2 \mathrm{~A}+\mathrm{B}$
4. Simplify: $\cos A\left[\begin{array}{cc}\cos A & \sin A \\ -\sin A & \cos A\end{array}\right]+\sin A\left[\begin{array}{cc}\sin A & -\cos A \\ \cos A & \sin A\end{array}\right]$
5. If $\left[\begin{array}{ccc}9 & -1 & 4 \\ -2 & 1 & 3\end{array}\right]=A+\left[\begin{array}{ccc}1 & 2 & -1 \\ 0 & 4 & 9\end{array}\right]$, then find the matrix $A$
6. If $A=\left[\begin{array}{ccc}2 & -1 & 0 \\ -2 & 1 & 1\end{array}\right]$ and $B=\left[\begin{array}{ccc}3 & -1 & 2 \\ -2 & 1 & -3\end{array}\right]$, then find the matrix $C$ : $5 A+3 B+2 C$ is a null matrix.
7. Find the matrices $X$ and $Y$ if $X+Y=\left[\begin{array}{ll}7 & 0 \\ 2 & 5\end{array}\right]$ and $X-Y=\left[\begin{array}{ll}3 & 0 \\ 0 & 3\end{array}\right]$
8. If $X$ and $Y$ are two square matrices if order2, find $X$ and $Y$ if $2 X+3 Y=\left[\begin{array}{ll}2 & 3 \\ 4 & 0\end{array}\right]$ and $3 X+2 Y=\left[\begin{array}{cc}-2 & 2 \\ 1 & 5\end{array}\right]$

If $A$ and $B$ are two matrices, then product $A B$ is defined if and only if the numbers of columns in $A$ is equal to number of rows in $B$.
i.e. Number of columns of second matrix must be equal to number of rows of first matrix.

Note: The result will have the same number of rows as the 1 st matrix, and the same number of columns as the 2 nd matrix.

## ASSIGNMENT-4

1. If $A=\left[\begin{array}{lll}2 & 1 & 3 \\ 3 & 2 & 1\end{array}\right]$ and $B=\left[\begin{array}{ll}4 & 3 \\ 1 & 2\end{array}\right]$, find $A B$ and $B A$ (if exist)
2. Find the product of following:
i) $\left[\begin{array}{cc}0 & 2 \\ -2 & -5\end{array}\right] \cdot\left[\begin{array}{cc}6 & -6 \\ 3 & 0\end{array}\right]$

$$
\begin{gathered}
{\left[\begin{array}{cc}
6 & 0 \\
-27 & 12
\end{array}\right]} \\
{\left[\begin{array}{cc}
-30 & 24 \\
15 & -12
\end{array}\right]} \\
{\left[\begin{array}{cc}
-5 & -10 \\
8 & 13
\end{array}\right]}
\end{gathered}
$$

ii) $\left[\begin{array}{c}6 \\ -3\end{array}\right]\left[\begin{array}{ll}-5 & 4\end{array}\right]$
iii) $\left[\begin{array}{cc}-5 & -5 \\ -1 & 2\end{array}\right] \cdot\left[\begin{array}{cc}-2 & -3 \\ 3 & 5\end{array}\right]$
iv) $\left[\begin{array}{cc}0 & 5 \\ -3 & 1 \\ -5 & 1\end{array}\right] \cdot\left[\begin{array}{cc}-4 & 4 \\ -2 & -4\end{array}\right]$.
$\left[\begin{array}{cc}-10 & -20 \\ 10 & -16 \\ 18 & -24\end{array}\right]$
v) $\left[\begin{array}{lll}5 & 3 & 5 \\ 1 & 5 & 0\end{array}\right] \cdot\left[\begin{array}{cc}-4 & 2 \\ -3 & 4 \\ 3 & -5\end{array}\right]$.
$\left[\begin{array}{ll}-14 & -3 \\ -19 & 22\end{array}\right]$
vi) $\left[\begin{array}{cc}-4 & 2 \\ -3 & 4 \\ 3 & -5\end{array}\right] \cdot\left[\begin{array}{ccc}5 & 3 & 5 \\ 1 & 5 & 0\end{array}\right]$.

$$
\left[\begin{array}{ccc}
-18 & -2 & -20 \\
-11 & 11 & -15 \\
10 & -16 & 15
\end{array}\right]
$$

2. If $A=\left[\begin{array}{lll}1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0\end{array}\right]$ and $B=\left[\begin{array}{ccc}-1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4\end{array}\right]$ show that $A B \neq B A$
3. Compute the product $A B$ and $B A$ where $A=\left[\begin{array}{ccc}2 & -1 & 3 \\ 3 & 2 & 4\end{array}\right]$ and $B=\left[\begin{array}{cc}-1 & 2 \\ 2 & -4 \\ 3 & -5\end{array}\right]$
4. find $A^{2}$, if i) $A=\left[\begin{array}{ll}1 & 2 \\ 3 & 4\end{array}\right]$ ii) $A=\left[\begin{array}{ccc}-1 & 1 & 2 \\ 3 & 0 & 1 \\ 2 & -3 & 4\end{array}\right]$ iii) $A=\left[\begin{array}{lll}5 & 3 & 5 \\ 1 & 5 & 0\end{array}\right]$.
5. If $A=\left[\begin{array}{cc}\cos \emptyset & -\sin \emptyset \\ \sin \emptyset & \cos \emptyset\end{array}\right]$, Find $A^{2}$.
6. if $\left[\begin{array}{ll}3 & 4 \\ 2 & x\end{array}\right]\left[\begin{array}{l}x \\ 1\end{array}\right]=\left[\begin{array}{l}19 \\ 12\end{array}\right]$, find the value of x .
7. $\mathrm{A}=\left[\begin{array}{cc}a b & b^{2} \\ -a^{2} & -a b\end{array}\right]$, evaluate $\mathrm{A}^{2}$.
8. If $x\left[\begin{array}{l}2 \\ 3\end{array}\right]+y\left[\begin{array}{c}-1 \\ 1\end{array}\right]=\left[\begin{array}{c}10 \\ 5\end{array}\right]$
9. If $\left[\begin{array}{ll}1 & 0 \\ y & 5\end{array}\right]+2\left[\begin{array}{cc}x & 0 \\ 1 & -2\end{array}\right]=I_{2}$, find $x$ and $y$.
10. Given $3\left[\begin{array}{cc}x & y \\ z & w\end{array}\right]=\left[\begin{array}{cc}x & 6 \\ -1 & 2 w\end{array}\right]+\left[\begin{array}{cc}4 & x+y \\ z+w & 3\end{array}\right]$, find the values of $\mathrm{x}, \mathrm{y}, \mathrm{z}$ and w .
11. Find the value of x , if $\left[\begin{array}{lll}1 & x & 1\end{array}\right]\left[\begin{array}{ccc}1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2\end{array}\right]\left[\begin{array}{l}1 \\ 2 \\ x\end{array}\right]=0$.
12. If $\mathrm{A}=\left[\begin{array}{ll}3 & -2 \\ 4 & -2\end{array}\right]$ and $I=\left[\begin{array}{ll}1 & 0 \\ 0 & 1\end{array}\right]$, find k so that $\mathrm{A}^{2}=\mathrm{kA}-2 \mathrm{I}$.
13. If $\mathrm{F}(\mathrm{x})=\left[\begin{array}{ccc}\cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1\end{array}\right]$, show that $\mathrm{F}(\mathrm{x}) . \mathrm{F}(\mathrm{y})=\mathrm{F}(\mathrm{x}+\mathrm{y})$
14. Show that the matrix $A=\left[\begin{array}{ll}2 & 3 \\ 1 & 2\end{array}\right]$ satisfies the equation $A^{2}-4 A+I=0$, where $I$ is $2 \times 2$ identity matrix and O is $2 \times 2$ zero matrix.
15. If $\mathrm{A}=\left[\begin{array}{cc}0 & -\tan \frac{\alpha}{2} \\ \tan \frac{\alpha}{2} & 0\end{array}\right]$ and I is the identity matrix of order 2 , Show that $\mathrm{I}+\mathrm{A}=(\mathrm{I}-\mathrm{A})\left[\begin{array}{cc}\cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha\end{array}\right]$
16. Let $\mathrm{A}=\left[\begin{array}{cc}2 & -4 \\ 7 & 8\end{array}\right]$, $\mathrm{B}=\left[\begin{array}{ll}5 & 2 \\ 7 & 4\end{array}\right]$ and $\mathrm{C}=\left[\begin{array}{ll}2 & 5 \\ 3 & 8\end{array}\right]$. Find a matrix D such that $\mathrm{CD}-\mathrm{AB}=\mathrm{O}$.
17. Let the two matrices $A$ and $B$ be given by $A=\left[\begin{array}{ccc}1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2\end{array}\right]$ and $B=\left[\begin{array}{ccc}2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5\end{array}\right]$.

Verify $A B=B A=61$
11. Find the matrix $X$ such that $\left(\begin{array}{cc}2 & -1 \\ 0 & 1 \\ -2 & 4\end{array}\right) X=\left(\begin{array}{ccc}-1 & -8 & -10 \\ 3 & 4 & 0 \\ 10 & 20 & 10\end{array}\right)$
12. A trust fund has Rs 30000 to invest in two bond first pays $5 \%$ interest per year and second pays $7 \%$ using matrix multiplication determine how to divide Rs 30000 among two type of boards if trust must obtain annual interest of Rs 2000.
13. In a department store a customer $x$ purchases 2 packets of tea, 4 kg of rice and 5 dozen oranges. Customer y purchases 1 packet of tea .5 kg rice and 24 oranges. Price on 1 pack of tea is Rs $54,1 \mathrm{~kg}$ rice is Rs 22 and that of one dozen orange is Rs 24 . Use matrix multiplication method and calculate each individual bill.
14. A manufacturer produces three products $\mathrm{x}, \mathrm{y}, \mathrm{z}$ which he sells in two markets .

Annual sales one indicated

| Market | Produces |  |  |
| :--- | :--- | :--- | :--- |
|  | X | y | $z$ |
| I | 10000 | 2000 | 18000 |
| II | 6000 | 20000 | 8000 |

Of unit sale price of $x, y$ and $z$ are rs 2.50 ,rs 1.50 and $r s 1.00$ respectively find the total revenue in each market using matrices.
15. If $A=\left[\begin{array}{lll}1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3\end{array}\right]$, prove that: $A^{3}-6 A^{2}+7 A+2 \mid=0$
16. If $A=\left[\begin{array}{cc}1 & -1 \\ -1 & 1\end{array}\right]$, show that $A^{3}=4 A$
17. Find the value of $x$ for which $\left[\begin{array}{lll}x & 4 & 1\end{array}\right]\left[\begin{array}{lll}2 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & 2 & 4\end{array}\right]\left[\begin{array}{l}x \\ 3 \\ 1\end{array}\right]=0$

The matrix obtained by interchanging the rows and columns of a matrix $A$ is called transpose of the matrix and is denoted by $\mathrm{A}^{\top}$ or $\mathrm{A}^{\prime}$.

If $A=\left[\begin{array}{lll}2 & 3 & 4 \\ 1 & 6 & 5\end{array}\right]$ then $\mathrm{A}^{\prime}=\left[\begin{array}{ll}2 & 1 \\ 3 & 6 \\ 4 & 5\end{array}\right]$

## Properties of transpose of the matrices

(i) $\quad\left(A^{\prime}\right)^{\prime}=A$
(ii) $(\mathrm{kA})^{\prime}=\mathrm{kA} /$ where k is any constant
(iii) $(A+B)^{\prime}=A^{\prime}+B^{\prime}$.
(iv) $\quad(A B)^{\prime}=B^{\prime} A^{\prime}$.

Eg. If $A=\left[\begin{array}{lll}2 & 3 & 4 \\ 1 & 6 & 5\end{array}\right]$ and $B=\left[\begin{array}{ccc}1 & 3 & 2 \\ 2 & 3 & -3\end{array}\right]$, verify that
(i) $\quad\left(A^{\prime}\right)^{\prime}=A$
(ii) $(\mathrm{kA})^{\prime}=\mathrm{kA} /$ where k is any constant
(iii) $(A+B)^{\prime}=A^{\prime}+B^{\prime}$.

Eg. If $A=\left[\begin{array}{cc}2 & -4 \\ 7 & 8\end{array}\right]$ and $B=\left[\begin{array}{ll}5 & 2 \\ 7 & 4\end{array}\right]$ verify that $(A B)^{\prime}=B^{\prime} A^{\prime}$.
Verify that $(A B)^{\prime}=B^{\prime} A^{\prime}$.
i) $\quad A=\left[\begin{array}{c}2 \\ -1 \\ 3\end{array}\right]$ and $B=\left[\begin{array}{lll}1 & 4 & -2\end{array}\right]$
ii) If $\mathrm{A}=\left[\begin{array}{ccc}1 & 1 & 0 \\ -2 & 3 & 4 \\ 0 & 1 & 2\end{array}\right]$ and $\mathrm{B}=\left[\begin{array}{lll}1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3\end{array}\right]$

## SYMMETRIC AND SKEW SYMMETRIC MATRICES

A square matrix $A$ is said to be symmetric if $A^{\prime}=A$, i.e. $a_{i j}=a_{\mathrm{ji}}$ for all possible values of i and j
For eg: $A=\left[\begin{array}{ccc}\sqrt{3} & 5 & -2 \\ 5 & 14 & 6 \\ -2 & 6 & 10\end{array}\right]$ is a symmetric matrix as $A^{\prime}=A$
A square matrix $A$ is said to be skew symmetric if $A^{\prime}=-A$, i.e. $a_{i j}=-a_{j i}$ for all possible values of $i$ and $j$ and $a_{i \mathrm{i}}=0$
For eg: $A=\left[\begin{array}{ccc}0 & 5 & 2 \\ -5 & 0 & 6 \\ -2 & -6 & 0\end{array}\right]$ is a symmetric matrix as $A^{\prime}=-A$
Note: diagonals elements of a skew symmetric matrix are zero.
Some important results:

1. For any square matrix $A, A+A^{\prime}$ is always symmetric. $A=\left[\begin{array}{ll}1 & 2 \\ 3 & 4\end{array}\right]$
2. For any square matrix $A, A-A^{\prime}$ is always skew symmetric. $A=\left[\begin{array}{cc}2 & 3 \\ 3 & -4\end{array}\right]$
3. Every square matrix can be uniquely expressed as sum of symmetric and skew symmetric matrix.

## Assignment-6

1. Express the matrix $A=\left[\begin{array}{ccc}2 & 2 & 4 \\ -4 & 2 & -4 \\ 2 & -10 & 5\end{array}\right]$ as a sum of symmetric and skew symmetric matrix.
2. Express the matrix $\left[\begin{array}{ccc}1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 5\end{array}\right]$ as sum of symmetric and skew symmetric matrix.
3. Find the values of $x, y, z$ if $A=\left[\begin{array}{ccc}0 & 2 y & z \\ x & y & -z \\ x & -y & z\end{array}\right]$ satisfies the eqn $A^{\prime} A=I$.
4. Find x if $\left[\begin{array}{lll}\mathrm{x} & 4 & -1\end{array}\right]\left[\begin{array}{ccc}2 & 1 & -1 \\ 1 & 0 & 0 \\ 2 & 2 & 4\end{array}\right]\left[\begin{array}{lll}\mathrm{x} & 4 & -1\end{array}\right]^{\mathrm{T}}=0$.
5. $A=\left[\begin{array}{cc}3 & 2 \\ -1 & 1\end{array}\right], B=\left[\begin{array}{cc}-1 & 0 \\ 2 & 5 \\ 3 & 4\end{array}\right]$, find $(B A)^{T}$.
6. If $A$ and $B$ are symmetric matrices of same order, write whether $A B-B A$ is symmetric or skew symmetric.
7. $A=\left[\begin{array}{cc}\cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha\end{array}\right]$, then write whether $A A^{T}$ is $O$ or $I$.
8. Find $x$, if $\left(\begin{array}{cc}5 & 3 x \\ 2 y & z\end{array}\right)=\left(\begin{array}{cc}5 & 4 \\ 12 & 6\end{array}\right)^{T}$
9. Define symmetric matrix and skew symmetric matrix. Give an example of each.
