CLASS XII ALGEBRA (10 MARKS)

MATHEMATICS

MATRICES

MATRIX: A matrix is an ordered rectangular array(i.e. arrangements of numbers or functions) such as

 $\mathsf{A} = \begin{bmatrix} 2 & \cos x & \sqrt{3} \\ 1.1 & 4 & 0 \end{bmatrix}$

Matrices are represented by capital letter A,B,C..etc.

The numbers or functions in a matrix are called elements of the matrix. The elements in the above example are 2, cosx, $\sqrt{3}$, 1.1, 4 and 0.

A horizontal line of elements is called **row** of the matrix and the vertical line is called **column** of the matrix. The matrix in the above example has two rows and three columns.

ORDER OF THE MATRIX: A matrix having m rows and n columns is called a matrix of order m×n.

The matrix in the above example has order 2×3 .



In general , a m×n matrix is written as A= $\begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix}$

Matrix can be written in compact form as $A=[a_{ij}]_m \times_n$. the number of elements in $m \times n$ matrix is mn.

The element a_{ij} is in the ith row and jth column and it is called (I,j)th element.

TYPES OF MATRICES:

1. ROW MATRIX: A matrix having one row only.

Example A= $\begin{bmatrix} -7 & \sqrt{2} & 1 \end{bmatrix}$

2. COLUMN MATRIX: A matrix having one column only.

Example: B= 4

3. SQUARE MATRIX: A matrix is said to be square if it has same number of rows and columns.

For example: $A = \begin{bmatrix} 1 & 5 \\ -9 & 2 \end{bmatrix}$ or $B = \begin{bmatrix} 1 & 1 & 3 \\ 8 & 0 & -5 \\ 7 & 8 & 99 \end{bmatrix}$

The elements a₁₁, a₂₂, a₃₃.... are called diagonal elements . The line along which the diagonal elements lie is called principal diagonal.

CLASS XII MATHEMATICS 4. DIAGONAL MATRIX: it is a square matrix in which all diagonal elements are non zero and rest all zeros. Ex: $C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 9 \end{bmatrix} = diag (1,6,9)$ 5. SCALAR MATRIX: A square matrix is called scalar if diagonals elements are same and rest all zeros. For ex: $X = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ In other words, a scalar matrix is a diagonal matrix with identical diagonal elements. 6. IDENTITY MATRIX (Unit Matrix): A square matrix is called identity matrix or unit matrix if all the non diagonal elements are zero and diagonals elements are unity. The identity matrix of order n is denoted by In. For ex: $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ are identity matrices of order 2 and 3. 7. NULL/ZERO MATRIX: A matrix whose all elements are zero is called null matrix or zero matrix .it is denoted by O. For ex : $O = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$. It can be of any order. 8. **UPPER TRIANGULAR MATRIX:** A square matrix A is said to be an upper triangular matrix if all the elements lying below the main diagonal are zero. ie. If $a_{ij}=0$ for all i>j. for ex: A= $\begin{bmatrix} 1 & 5 & 5 \\ 0 & 4 & 6 \\ 0 & 0 & 9 \end{bmatrix}$. 9. LOWER TRIANGULAR MATRIX: A square matrix A is said to be an lower triangular matrix if all the elements lying above the principal diagonal are zero. ie. If $a_{ij}=0$ for all i < j. for ex: A= $\begin{bmatrix} 1 & 0 & 0 \\ 3 & 4 & 0 \\ 7 & 8 & 9 \end{bmatrix}$

MATHEMATICS

ASSIGNMENT-1

1. let $A = \begin{bmatrix} 3 & -7 & -3 \\ \sqrt{3} & 55 & 2.5 \end{bmatrix}$ be any matrix Write i) order of matrix ii) number of elements iii) elements a₁₂, a₃₂, a₂₁. If a matrix has 18 elements, what are the possible orders it can have? What if it has 11 elements? 2. construct a 3×4 matrix A=[a_{ij}], whose elements are given by $a_{ij} = \frac{|i-j|}{2}$ 3. Write the element a_{12} , a_{22} of matrix A= $[a_{ij}]$ whose elements a_{ij} are given by $a_{ij} = e^{2ix} sinjx$. 4. construct a 3×4 matrix whose elements are given by $a_{ij} = \begin{cases} i+j & if \ i \ge j \\ \frac{1}{2^{(i-1)}} & if \ i < j \end{cases}$ 5. Write the number of all possible matrices of order 2×3 with each entry 1 or 2. 6. 7. What is the number of all possible matrices of order 3×3 with each entry 9 or 8 ? write the number of all possible matrices of order 2×2 with each entry 2, 3, or 4. 8. 9. Give an example of : i) scalar matrix ii) diagonal matrix iii) row matrix which is also a column matrix 10. If $A = \begin{bmatrix} 3 & 2 & 0 \\ -1 & 1.5 & 4 \\ 7 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 22 & 8 \\ 7 & 2 & 5 \\ 8 & 0 & 2 \end{bmatrix}$, then find $a_{22}+b_{21}$. EQUALITY OF MATRICES: Two matrices A and B are said to be equal if their order is same and corresponding elements are equal i.e a $_{ij}=b_{ij}$. If A and B are equal matrices, then we write A=B. Otherwise we write A \neq B **ASSIGNMENT-2** 1. Find the values of x, y and z if $\begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$ 2. Find the value of x if $\begin{vmatrix} 3x + y & -y \\ 2y - x & 3 \end{vmatrix} = \begin{vmatrix} 1 & 2 \\ -5 & 3 \end{vmatrix}$ 3. Write the value of x-y+z from the equation: $\begin{bmatrix} x + y + z \\ x + z \\ y + z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$ Find the values of a and b if A=B, where A= $\begin{bmatrix} a+4 & 3b \\ 8 & -6 \end{bmatrix}$ and $B = \begin{bmatrix} 2a+2 & b^2+2 \\ 8 & b^2-5b \end{bmatrix}$ 4. 5. If $A = \begin{bmatrix} cos \phi & -sin \phi \\ sin \phi & cos \phi \end{bmatrix}$, then for what value of ϕ is A an identity matrix? 6. Find the values of a, b, c and d from the equation: $\begin{bmatrix} 2a+b & a-2b \\ 5c-d & 4c+3d \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 11 & 24 \end{bmatrix}$

MATHEMATICS OPERATIONS ON MATRICES

ADDITION OF TWO MATRICES: Let A = $[a_{ij}]$ and B= $[b_{ij}]$ be two matrices of same order, then we define sum of matrices A and B as A+B= $[a_{ij}+b_{ij}]$.

In simple words, A+B is a matrix obtained by adding the corresponding elements of A and B.

Let $A = \begin{bmatrix} 1 & 2 \\ -1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 1 \\ 0 & -2 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 3 & 5 \\ 1 & 0 & -2 \end{bmatrix}$ then $A + B = \begin{bmatrix} 5 & 3 \\ -1 & 0 \end{bmatrix}$ but

A+C and B+C are not defined as their order is not same.

NOTE: i) If A and B are not of same order, then A+B is not defined.

ii) Matrix addition is commutative: A+B= B+A

iii) Matrix addition is associative: A+(B+C)= (A+B)+C

<u>SUBTRACTION</u> OF TWO MATRICES: Let A = $[a_{ij}]$ and B= $[b_{ij}]$ be two matrices of same order, then we define difference of matrices A nd B as A-B= $[a_{ij}-b_{ij}]$.

In simple words, A-B is a matrix obtained by subtracting the corresponding elements of A and B.

NOTE: If A and B are not of same order, then A-B is not defined.

Let $A = \begin{bmatrix} 1 & 2 \\ -1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 1 \\ 0 & -2 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 3 & 5 \\ 1 & 0 & -2 \end{bmatrix}$ then $A - B = \begin{bmatrix} -3 & 1 \\ -1 & 4 \end{bmatrix}$

but A-C and B-C are not defined as their order is not same.

SCALAR MULTIPLICATION (multiplication of a matrix by a scalar): Let A =[a_{ij}] e a matrix of any order and let k be ay scalar, then we define multiplication of matrix A by scalar k as kA=[ka_{ij}]

In simple words, kA is a matrix obtained by multiplying each element of A by scalar k.

For ex: let $A = \begin{bmatrix} 1 & 2 \\ -1 & 2 \end{bmatrix}$. Then $2A = 2\begin{bmatrix} 1 & 2 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ -2 & 4 \end{bmatrix}$.

MATHEMATICS

ASSIGNMENT-3

1. If $A = \begin{bmatrix} -1 & 0 \\ 3 & 9 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -5 \\ -3 & 5 \end{bmatrix}$, then find A+B, A-B, 3A

2. If $A = \begin{bmatrix} -1 & 0 \\ 4 & 5 \end{bmatrix} and B = \begin{bmatrix} 0 & -5 \\ -3 & 7 \end{bmatrix}$, then find 3A-2B 3. If A= diag (-2,5,1) and B= diag(1,0,-4), then find 2A+B 4. Simplify: $\cos A \begin{bmatrix} cosA & sinA \\ -sinA & cosA \end{bmatrix} + sinA \begin{bmatrix} sinA & -cosA \\ cosA & sinA \end{bmatrix}$ 5. If $\begin{bmatrix} 9 & -1 & 4 \\ -2 & 1 & 3 \end{bmatrix} = A + \begin{bmatrix} 1 & 2 & -1 \\ 0 & 4 & 9 \end{bmatrix}$, then find the matrix A 6. If $A = \begin{bmatrix} 2 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -1 & 2 \\ -2 & 1 & -3 \end{bmatrix}$, then find the matrix C: 5A+3B+2C is a null matrix. 7. Find the matrices X and Y if X+Y= $\begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$ and X-Y = $\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$ 8. If X and Y are two square matrices if order2, find X and Y if 2X+3Y= $\begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$ and $3X+2Y=\begin{bmatrix} -2 & 2 \\ 1 & 5 \end{bmatrix}$

MATHEMATICS **MUTIPLICATION OF MATRICES**

If A and B are two matrices, then product AB is defined if and only if the numbers of columns in A is equal to number of rows in B.

i.e. Number of columns of second matrix must be equal to number of rows of first matrix.

Note: The result will have the same number of rows as the 1st matrix, and the same number of columns as the 2nd matrix.

ASSIGNMENT-4

1. If
$$A = \begin{bmatrix} 2 & 1 & 3 \\ 3 & 2 & 1 \end{bmatrix}$$
 and $B = \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix}$, find AB and BA (if exist)

2. Find the product of following:

	i) $\begin{bmatrix} 0 & 2 \\ -2 & -5 \end{bmatrix} \cdot \begin{bmatrix} 6 & -6 \\ 3 & 0 \end{bmatrix}$		$\begin{bmatrix} 6 \\ -27 \end{bmatrix}$	0 12]	
	ii) $\begin{bmatrix} 6 \\ -3 \end{bmatrix} \begin{bmatrix} -5 & 4 \end{bmatrix}$		$\begin{bmatrix} -30 \\ 15 \end{bmatrix}$	$^{24}_{-12}]$	
	iii) $\begin{bmatrix} -5 & -5 \\ -1 & 2 \end{bmatrix} \cdot \begin{bmatrix} -2 & -3 \\ 3 & 5 \end{bmatrix}$	$\begin{bmatrix} -5 \\ 8 \end{bmatrix}$	$^{-10}_{13}]$		
	iv) $\begin{bmatrix} 0 & 5 \\ -3 & 1 \\ -5 & 1 \end{bmatrix} \cdot \begin{bmatrix} -4 & 4 \\ -2 & -4 \end{bmatrix}$.		$\begin{bmatrix} -10\\10\\18\end{bmatrix}$	-20^{-} -16^{-} -24^{-}	
	v) $\begin{bmatrix} 5 & 3 & 5 \\ 1 & 5 & 0 \end{bmatrix} \cdot \begin{bmatrix} -4 & 2 \\ -3 & 4 \\ 3 & -5 \end{bmatrix}$.	$\begin{bmatrix} -14 \\ -19 \end{bmatrix}$	$\binom{-3}{22}$		
	vi) $\begin{bmatrix} -4 & 2 \\ -3 & 4 \\ 3 & -5 \end{bmatrix} \cdot \begin{bmatrix} 5 & 3 & 5 \\ 1 & 5 & 0 \end{bmatrix}$.		$\begin{bmatrix} -18\\ -11\\ 10 \end{bmatrix}$	-2 11 -16	-20 ⁻ -15 15
2.	If A= $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$ and B= $\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}$ show that AB \neq BA				
3.	Compute the product AB and BA where A= $\begin{bmatrix} 2 & -1 & 3 \\ 3 & 2 & 4 \end{bmatrix}$ and B= $\begin{bmatrix} -1 & 2 \\ 2 & -4 \\ 3 & -5 \end{bmatrix}$				
4.	find A ² , if i) A= $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ ii) A= $\begin{bmatrix} -1 & 1 & 2 \\ 3 & 0 & 1 \\ 2 & -3 & 4 \end{bmatrix}$ iii) A= $\begin{bmatrix} 5 & 3 & 5 \\ 1 & 5 & 0 \end{bmatrix}$.				
5.	If $A = \begin{bmatrix} cos \phi & -sin \phi \\ sin \phi & cos \phi \end{bmatrix}$, Find A^2 .				
6.	if $\begin{bmatrix} 3 & 4 \\ 2 & x \end{bmatrix} \begin{bmatrix} x \\ 1 \end{bmatrix} = \begin{bmatrix} 19 \\ 12 \end{bmatrix}$, find the value of x.				
7.	$A = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}, \text{ evaluate } A^2.$				

MATHEMATICS ASSIGNMENT-5

1. If $x \begin{bmatrix} 2 \\ 3 \end{bmatrix} + y \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$ 2. If $\begin{bmatrix} 1 & 0 \\ y & 5 \end{bmatrix} + 2 \begin{bmatrix} x & 0 \\ 1 & -2 \end{bmatrix} = I_2$, find x and y. 3. Given 3 $\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}$, find the values of x, y, z and w. 4. Find the value of x, if $\begin{bmatrix} 1 & x & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ r \\ r \end{bmatrix} = 0$. 5. If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find k so that $A^2 = kA - 2I$. 6. If $F(x) = \begin{bmatrix} cosx & -sinx & 0 \\ sinx & cosx & 0 \\ 0 & 0 & 1 \end{bmatrix}$, show that F(x).F(y) = F(x+y)7. Show that the matrix $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ satisfies the equation $A^2 - 4A + I = 0$, where I is 2 x 2 identity matrix and O is 2 x 2 zero matrix. 8. If A = $\begin{bmatrix} 0 & -tan\frac{\alpha}{2} \\ tan\frac{\alpha}{2} & 0 \end{bmatrix}$ and I is the identity matrix of order 2, Show that I + A = (I-A) $\begin{bmatrix} cos\alpha & -sin\alpha \\ sin\alpha & cos\alpha \end{bmatrix}$ 9. Let $A = \begin{bmatrix} 2 & -4 \\ 7 & 8 \end{bmatrix}$, $B = \begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}$. Find a matrix D such that CD - AB = O. 10. Let the two matrices A and B be given by $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$. Verify AB = BA = 6I11. Find the matrix X such that $\begin{pmatrix} 2 & -1 \\ 0 & 1 \\ 2 & 4 \end{pmatrix} X = \begin{pmatrix} -1 & -8 & -10 \\ 3 & 4 & 0 \\ 10 & 20 & 10 \end{pmatrix}$ 12. A trust fund has Rs 30000to invest in two bond first pays 5% interest per year and second pays 7% using matrix multiplication determine how to divide Rs 30000 among two type of boards if trust must obtain annual interest of Rs 2000.

- 13. In a department store a customer x purchases 2 packets of tea, 4 kg of rice and 5 dozen oranges. Customer y purchases 1 packet of tea .5 kg rice and 24 oranges. Price on 1 pack of tea is Rs 54, 1kg rice is Rs 22 and that of one dozen orange is Rs 24. Use matrix multiplication method and calculate each individual bill.
- 14. A manufacturer produces three products x, y, z which he sells in two markets .

MATHEMATICS TRANSPOSE OF A MATRIX

The matrix obtained by interchanging the rows and columns of a matrix A is called transpose of the matrix and is denoted by A^{T} or A^{\prime} .

If $A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 6 & 5 \end{bmatrix}$ then $A' = \begin{bmatrix} 2 & 1 \\ 3 & 6 \\ 4 & 5 \end{bmatrix}$

Properties of transpose of the matrices

(i)
$$(A')' = A$$

(ii) $(kA)' = kA'$ where k is any constant
(iii) $(A+B)' = A'+B'$.
(iv) $(AB)' = B'A'$.
Eg. If $A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 6 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 3 & -3 \end{bmatrix}$, verify that
(i) $(A')' = A$
(ii) $(kA)' = kA'$ where k is any constant
(iii) $(A+B)' = A'+B'$.
Eg. If $A = \begin{bmatrix} 2 & -4 \\ 7 & 8 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}$ verify that $(AB)' = B'A'$.
Verify that $(AB)' = B'A'$.
i) $A = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 & -2 \end{bmatrix}$

ii) If A =
$$\begin{bmatrix} 1 & 1 & 0 \\ -2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$$
 and B= $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$

MATHEMATICS

SYMMETRIC AND SKEW SYMMETRIC MATRICES

A square matrix A is said to be **symmetric** if $A^{\prime}=A$, i.e. $a_{ij}=a_{ji}$ for all possible values of i and j

For eg: A=
$$\begin{bmatrix} \sqrt{3} & 5 & -2 \\ 5 & 14 & 6 \\ -2 & 6 & 10 \end{bmatrix}$$
 is a symmetric matrix as A/=A

A square matrix A is said to be **skew symmetric** if $A^{/}=-A$, i.e. $a_{ij}=-a_{ji}$ for all possible values of i and j and $a_{ii}=0$

For eg: A=
$$\begin{bmatrix} 0 & 5 & 2 \\ -5 & 0 & 6 \\ -2 & -6 & 0 \end{bmatrix}$$
 is a symmetric matrix as A[/]= -A

Note: diagonals elements of a skew symmetric matrix are zero.

Some important results:

- **1.** For any square matrix A , A+A[/] is always symmetric. A= $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$
- 2. For any square matrix A , A-A' is always skew symmetric. $A = \begin{bmatrix} 2 & 3 \\ 3 & -4 \end{bmatrix}$
- 3. Every square matrix can be uniquely expressed as s sum of symmetric and skew symmetric matrix.

Assignment-6

1. Express the matrix $A = \begin{bmatrix} 2 & 2 & 4 \\ -4 & 2 & -4 \\ 2 & -10 & 5 \end{bmatrix}$ as a sum of symmetric and skew symmetric matrix. 2. Express the matrix $\begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 5 \end{bmatrix}$ as sum of symmetric and skew symmetric matrix. 3. Find the values of x, y, z if $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ satisfies the eqn A/A = I. 4. Find x if $\begin{bmatrix} x & 4 & -1 \end{bmatrix} \begin{bmatrix} 2 & 1 & -1 \\ 1 & 0 & 0 \\ 2 & 2 & 4 \end{bmatrix} \begin{bmatrix} x & 4 & -1 \end{bmatrix}^T = 0.$ 5. $A = \begin{bmatrix} 3 & 2 \\ -1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 0 \\ 2 & 5 \\ 3 & 4 \end{bmatrix}$, find (BA)^T. 6. If A and B are symmetric matrices of same order, write whether AB – BA is symmetric or skew symmetric. 7. $A = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$, then write whether AA^T is O or I.

- 8. Find x, if $\begin{pmatrix} 5 & 3x \\ 2y & z \end{pmatrix} = \begin{pmatrix} 5 & 4 \\ 12 & 6 \end{pmatrix}^T$
- 9. Define symmetric matrix and skew symmetric matrix. Give an example of each.