Class X Maths

FUN TIME

Akhila went to a fair in her village. She wanted to enjoy rides on the Giant Wheel and play Hoopla (a game in which you throw a ring on the items kept in a stall, and if the ring covers any object completely, you get it). The number of times she played Hoopla is half the number of rides she had on the Giant Wheel. If each ride costs Rs 3, and a game of Hoopla costs Rs 4, how would you find out the number of rides she had and how many times she played Hoopla, provided she spent Rs 20.

How to correlate this situation mathematically?

SOLUTION

Let the number of rides that Akhila had by x, and the number of times she played Hoopla by y.

Now the situation can be represented by the two equations:

- $y = \frac{1}{2}x$ (1) (The number of times she played Hoopla is half the number of rides she had on the Giant Wheel)
- $3x + 4y = 20$ (2) (If each ride costs Rs 3, and a game of Hoopla costs Rs 4 and she spent Rs 20.)

Linear equations in two variables

- Linear means power of the variable used is 1.
- Equation means algebraic expression with equal to sign.
- Variables mean unknown quantity.

GENERAL FORM OF A LINEAR EQUATION IN TWO VARIABLES

a x + b y + c = 0 ,where a,b and c are real numbers , a and b are not both zero (We often denote the condition a and b are not both zero by $a^2 + b^2 \neq 0$).

GENERAL FORM OF PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$,

where a_1, b_1, c_1, a_2, b_2, c_2 are all real numbers and

${a_1}^2 + {b_1}^2 \neq 0$, ${a_2}^2 + {b_2}^2 \neq 0$

SOLUTION OF AN EQUATION

A solution of such an equation is a

pair of values, one for *x* and the other for *y*, which makes the two sides of the equation equal

e.g. $x + y = 3$

(1,2) ,(3,0)

Can we find the solutions of this pair of equations?

- $y = \frac{1}{2}x \qquad (1)$
- $3x + 4y = 20 \qquad (2)$

Solutions to equations :

$$y = \frac{1}{2}x \qquad (1)$$

$$3x + 4y = 20 \qquad (2)$$

x	0	2	4
y	o	1	2

x	0	4	6
y	5	2	0.5

Geometrically, what does this mean?

- It means that the point (4,2) lies on the line
- representing the equation $y = \frac{1}{2}x$, and the point (4,2) also lies on the line representing the equation $3x + 4y = 20$.
- So, **every solution of the equation is a point on the line representing it**.
- But the point (2, 1) which satisfy $y = \frac{1}{2}x$ *is not satisfying*

  $3x + 4y = 20$

What do they look like geometrically?

- The geometrical (i.e., graphical) representation of a linear equation in two variables is a straight line.

What a pair of linear equations in two variables will look like, geometrically?

- There will be two straight lines, both to be considered together.
- If two lines in a plane are drawn , only one of the following three possibilities can happen:

  (i) The two lines will intersect at one point.

  (ii) The two lines will not intersect, i.e., they are parallel.

  (iii) The two lines will be coincident.

PAIR OF EQUATIONS

CONSISTENT

UNIQUE SOLUTION

- $\frac{a1}{a2} \neq \frac{b1}{b2}$
- Intersecting lines (interdependent)

INFINITE SOLUTIONS

- $\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2}$
- Coincident lines (DEPENDENT)

INCONSISTENT

NO SOLUTION

- $\frac{a1}{a2} = \frac{b1}{b2} \neq \frac{c1}{c2}$
- Parallel lines (Not dependent)

CONDITIONS FOR PAIR OF EQUATIONS

Sl No.	Pair of lines	$\frac{a_1}{a_2}$	$\frac{b_1}{b_2}$	$\frac{c_1}{c_2}$	Compare the ratios	Graphical representation	Algebraic interpretation
1.	$x-2y=0$ $3x+4y-20=0$	$\frac{1}{3}$	$\frac{-2}{4}$	$\frac{0}{-20}$	$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Intersecting lines	Exactly one solution (unique)
2.	$2x+3y-9=0$ $4x+6y-18=0$	$\frac{2}{4}$	$\frac{3}{6}$	$\frac{-9}{-18}$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	Coincident lines	Infinitely many solutions
3.	$x+2y-4=0$ $2x+4y-12=0$	$\frac{1}{2}$	$\frac{2}{4}$	$\frac{-4}{-12}$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	Parallel lines	No solution

Ex 3.2 Q2

Find out whether the lines representing the following pairs of linear equations intersect at a point, are parallel or coincident:

(i) $5x - 4y + 8 = 0$

$7x + 6y - 9 = 0$

(ii) $9x + 3y + 12 = 0$

$18x + 6y + 24 = 0$

(iii) $6x - 3y + 10 = 0$

$2x - y + 9 = 0$

Ex 3.2 Q 3

Find out whether the following pair of linear equations are consistent, or inconsistent.

(i) $3x + 2y = 5$; $2x - 3y = 7$

(ii) $2x - 3y = 8$; $4x - 6y = 9$

(iii) $\frac{3}{2}x + \frac{5}{3}y = 7$; $9x - 10y = 14$

(iv) $5x - 3y = 11$; $-10x + 6y = -22$

(v) $\frac{4}{3}x + 2y = 8$; $2x + 3y = 12$

Ex 3.2 Q 4

Find out whether the following pair of linear equations are consistent, or inconsistent

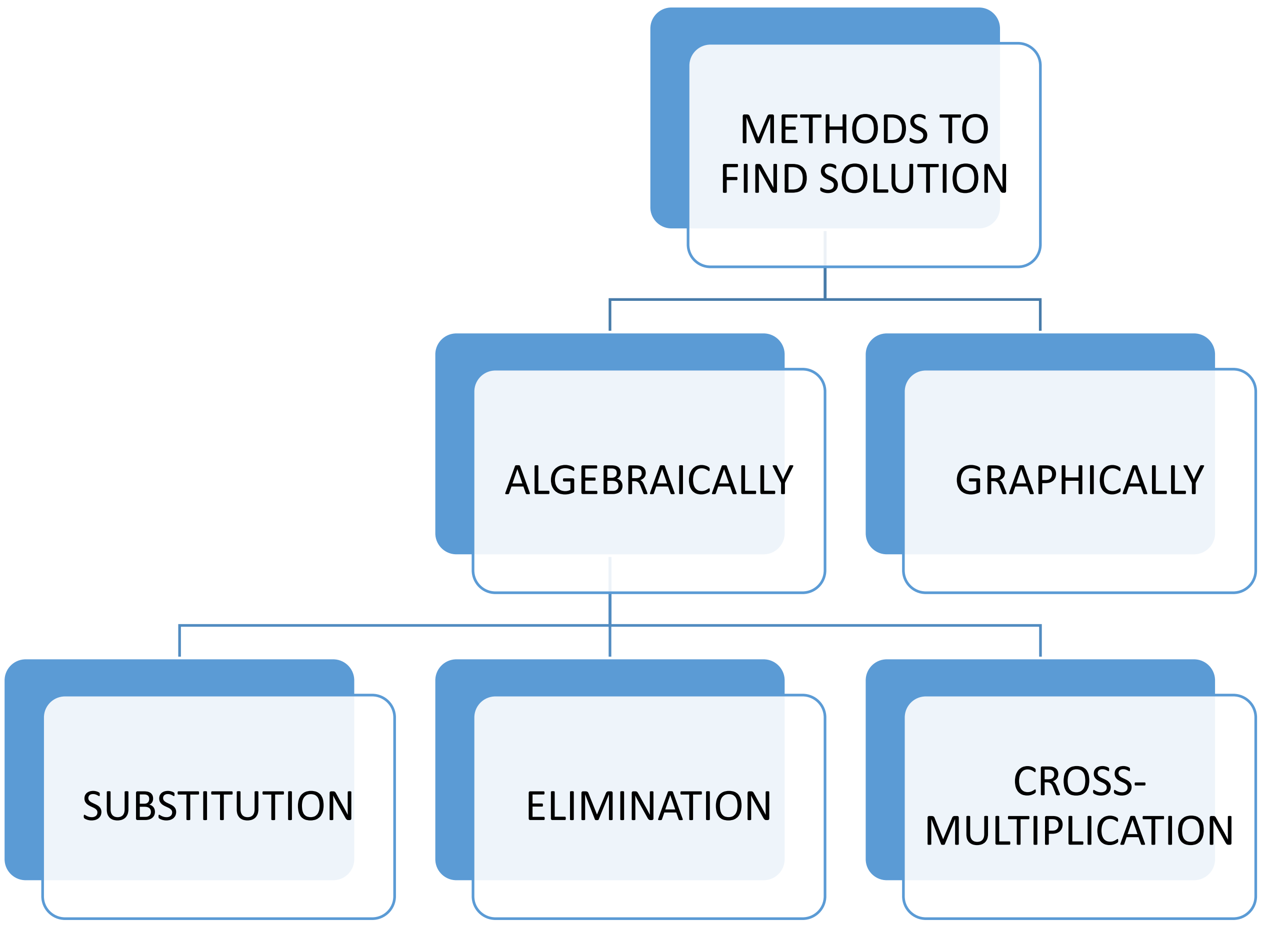
(i) $x + y = 5$, $2x + 2y = 10$

(ii) $x - y = 8$, $3x - 3y = 16$

(iii) $2x + y - 6 = 0$, $4x - 2y - 4 = 0$

(iv) $2x - 2y - 2 = 0$, $4x - 4y - 5 = 0$

SOLUTIONS TECHNIQUE



SUBSTITUTION METHOD

Let us take

$$3x + 2y = 1\ 2 \qquad (1)$$

$$x + y = 5 \qquad (2)$$

Solution :

- **Step 1 :** We pick either of the equations and write one variable in terms of the other.

Let us consider the Equation (2) :

$$x + y = 5$$

and write it as $x = 5 - y$ (3)

- **Step 2 :** Substitute the value of $x = 5 - y$ in Equation (1)

i.e $3x + 2y = 1\ 2$ (1)

We get

$3\ (5 - y\ ) + 2y = 12$

i.e., $15 - 3y + 2y = 12$

i.e., $-y = 12 - 15$

i.e , $-y = -3$

Therefore, $y = 3$

- **Step 3 :** Substituting this value of $y = 3$ in Equation (3) i.e $x = 5 - y$

  we get

  $x = 5 - 3$

  $x = 2$

Therefore, the solution is $x = 2$, $y = 3$ or $(2, 3)$

ELIMINATION METHOD

Eg 12 NCERT

$$x + 3y = 8 \qquad (1)$$

$$4x + 6y = 10 \qquad (2)$$

Solution :

- **Step 1 :** Multiply Equation (1) by 2 and

  Equation (2) by 1 to make the coefficients of *x* equal.

Then we get the equations as :

$$2x + 6y = 16 \qquad (3)$$

$$4x + 6y = 10 \qquad (4)$$

Subtracting equation (4) from (3)

$$2x + 6y = 16 \qquad (3)$$

$$4x + 6y = 10 \qquad (4)$$

- - -

$$-2x = 6$$

$$x = -3$$

Substitute x= -3 in any of equation ,say it as (1) $x + 3y = 8$

$$-3 + 3y = 8$$

$$3y = 11$$

$$y = \frac{11}{3}$$

- x = -3 and y = $\frac{11}{3}$ is the solution.
- H.W : Do question 1 (i ,ii) of Ex 3.3 by both methods in P.C