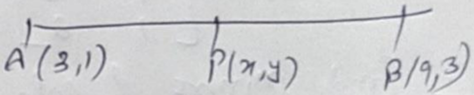
23/3/20 (Tuesday)

1) Find the equation of line joining the points $A(3,1)$ & $B(9,3)$ using determinants.

Sol: Let $P(x,y)$ be any point on the line AB

$\therefore$ A, B & P are collinear




$$\Rightarrow \frac{1}{2}\begin{vmatrix} 3 & 1 & 1 \\ x & y & 1 \\ 9 & 3 & 1 \end{vmatrix} = 0$$

$$\Rightarrow 3(y-3) - 1(x-9) + 1(3x-9y) = 0$$

$$\Rightarrow 3y - 9 - x + 9 + 3x - 9y = 0$$

$$\Rightarrow \boxed{2x - 6y = 0}$$

2) Find K if area of $\Delta$ with vertices $(K,4)$, $(2,-6)$ & $(5,4)$ is 35 sq units.

[-2, 12]

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## MINORS & COFACTORS OF A MATRIX:

Minor of an element $a_{ij}$ of a determinant is the determinant obtained by deleting its $i$th row & $j$th column in which the element $a_{ij}$ lies. Minor of an element is denoted by $M_{ij}$.

eg: $\Delta = \begin{vmatrix} 1 & -2 \\ 4 & 3 \end{vmatrix}$ find minors.

$M_{11} = |3| = 3$, $M_{12} = 4$, $M_{21} = -2$, $M_{22} = 1$.

eg: $\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$ find all minors.

$M_{11} = \begin{vmatrix} 5 & 6 \\ 8 & 9 \end{vmatrix} = -3$ $\quad M_{12} = \begin{vmatrix} 4 & 6 \\ 7 & 9 \end{vmatrix} = -6$ $\quad M_{13} = \begin{vmatrix} 4 & 5 \\ 7 & 8 \end{vmatrix} = -3$

$M_{21} = \begin{vmatrix} 2 & 3 \\ 8 & 9 \end{vmatrix} = -6$ $\quad M_{22} = \begin{vmatrix} 1 & 3 \\ 7 & 9 \end{vmatrix} = -12$ $\quad M_{23} = \begin{vmatrix} 1 & 2 \\ 7 & 8 \end{vmatrix} = -6$

$M_{31} = \begin{vmatrix} 2 & 3 \\ 5 & 6 \end{vmatrix} = -3$ $\quad M_{32} = \begin{vmatrix} 1 & 3 \\ 4 & 6 \end{vmatrix} = -6$ $\quad M_{33} = \begin{vmatrix} 1 & 2 \\ 4 & 5 \end{vmatrix} = -$

Cofactors: Cofactor of an element $a_{ij}$ denoted by $A_{ij}$ is defined by $\boxed{A_{ij} = (-1)^{i+j} M_{ij}}$

$A_{11} = M_{11}$
$A_{12} = -M_{12}$
$A_{21} = -M_{21}$
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Q: find minors & cofactors of $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$

$M_{11} = 4 \quad M_{12} = 3 \quad M_{21} = -1 \quad M_{22} = 2$

$A_{11} = 4, \quad A_{12} = -3, \quad A_{21} = 1, \quad A_{22} = 2$

Cofactors $A = \begin{bmatrix} 4 & -3 \\ 1 & 2 \end{bmatrix}$

Q: find minors & cofactors of $\Delta = \begin{vmatrix} 3 & -2 & 1 \\ 4 & 6 & 5 \\ 2 & -1 & 7 \end{vmatrix}$

$M_{11} = \begin{vmatrix} 6 & 5 \\ -1 & 7 \end{vmatrix} = 47, \quad M_{12} = \begin{vmatrix} 4 & 5 \\ 2 & 7 \end{vmatrix} = 18, \quad M_{13} = \begin{vmatrix} 4 & 6 \\ 2 & -1 \end{vmatrix} = -16$

$M_{21} = \begin{vmatrix} -2 & 1 \\ -1 & 7 \end{vmatrix} = -13, \quad M_{22} = \begin{vmatrix} 3 & 1 \\ 2 & 7 \end{vmatrix} = 19, \quad M_{23} = \begin{vmatrix} 3 & -2 \\ 2 & -1 \end{vmatrix} = 1$

$M_{31} = \begin{vmatrix} -2 & 1 \\ 6 & 5 \end{vmatrix} = -16 \quad M_{32} = \begin{vmatrix} 3 & 1 \\ 4 & 5 \end{vmatrix} = 11 \quad M_{33} = \begin{vmatrix} 3 & -2 \\ 4 & 6 \end{vmatrix} = 26$

$A_{11} = 47 \quad A_{12} = -18 \quad A_{13} = -16$

$A_{21} = 13 \quad A_{22} = 19 \quad A_{23} = -1$

$A_{31} = -16 \quad A_{32} = -11 \quad A_{33} = 26$

$$\text{Cofactors} = \begin{bmatrix} 47 & -18 & -16 \\ 13 & 19 & -1 \\ -16 & -11 & 26 \end{bmatrix}$$

Q: find the cofactors of the elements of the third row of the determinant $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$ and verify that

$a_{11} A_{31} + a_{12} A_{32} + a_{13} A_{33} = 0$

Sol $a_{11} = 2 \quad a_{12} = -3 \quad a_{13} = 5$

$M_{31} = \begin{vmatrix} -3 & 5 \\ 0 & 4 \end{vmatrix} = -12 \rightarrow A_{31} = -12$

$M_{32} = \begin{vmatrix} 2 & 5 \\ 6 & 4 \end{vmatrix} = -22 \rightarrow A_{32} = 22$

$M_{33} = \begin{vmatrix} 2 & -3 \\ 6 & 0 \end{vmatrix} = 18 \rightarrow M_{33} = 18$

$LHS = a_{11} A_{31} + a_{12} A_{32} + a_{13} A_{33} = 2(-12) + (-3)(22) + 5(18)$

$= -24 - 66 + 90$

$= 0.$

## Adjoint of a Matrix:

The adjoint of a square matrix $A$ is defined as the transpose of the matrix $A_{ij}$, where $A_{ij}$ is the cofactor matrix ie $\operatorname{adj}A = (\text{coff})^T$.

**eg 23**: $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$ find adj A.

$M_{11} = 4 \quad M_{12} = 1 \quad M_{21} = 3 \quad M_{22} = 2$

$A_{11} = 4 \quad A_{12} = -1 \quad A_{21} = -3 \quad A_{22} = 2$

$\text{cof} = \begin{bmatrix} 4 & -1 \\ -3 & 2 \end{bmatrix}$

$\operatorname{Adj}A = (\text{cof})^T = \begin{bmatrix} 4 & -3 \\ -1 & 2 \end{bmatrix}$

**Note**: The adjA can also be obtained by interchanging $a_{11}$ & $a_{22}$ and by changing signs of $a_{12}$ & $a_{21}$ ie

If $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$ then $\operatorname{adj}A = \begin{bmatrix} 4 & -3 \\ -1 & 2 \end{bmatrix}$

Sign change  interchange

**Result**: If $A$ be any square matrix of order $n$, then

$$\boxed{A(\operatorname{adj}A) = (\operatorname{adj}A)A = |A|\, I}$$

**Que**: $A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$ Verify $A(\operatorname{adj}A) = (\operatorname{adj}A)A = |A| I_3$

$|A| = 1\begin{vmatrix} 0 & -2 \\ 0 & 3 \end{vmatrix} + 1\begin{vmatrix} 3 & -2 \\ 1 & 3 \end{vmatrix} + 2\begin{vmatrix} 3 & 0 \\ 1 & 0 \end{vmatrix}$

$= 1(0) + 1(11) + 2(0) = 11.$

$M_{11} = \begin{vmatrix} 0 & -2 \\ 0 & 3 \end{vmatrix} = 0, \quad M_{12} = \begin{vmatrix} 3 & -2 \\ 1 & 3 \end{vmatrix} = 11, \quad M_{13} = \begin{vmatrix} 3 & 0 \\ 1 & 0 \end{vmatrix} = 0$

$M_{21} = \begin{vmatrix} -1 & 2 \\ 0 & 3 \end{vmatrix} = -3, \quad M_{22} = \begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix} = 1, \quad M_{23} = \begin{vmatrix} 1 & -1 \\ 1 & 0 \end{vmatrix} = 1$

$M_{31} = \begin{vmatrix} -1 & 2 \\ 0 & -2 \end{vmatrix} = 2, \quad M_{32} = \begin{vmatrix} 1 & 2 \\ 3 & -2 \end{vmatrix} = -8, \quad M_{33} = \begin{vmatrix} 1 & -1 \\ 3 & 0 \end{vmatrix} = 3$

$A_{11} = 0, \quad A_{12} = -11, \quad A_{13} = 0$

$A_{21} = 3, \quad A_{22} = 1, \quad A_{23} = -1$

$A_{31} = 2, \quad A_{32} = 8, \quad A_{33} = 3$

Cofactors $= \begin{bmatrix} 0 & -11 & 0 \\ 3 & 1 & -1 \\ 2 & 8 & 3 \end{bmatrix}$

$$\text{adj}A = (\text{coff})' = \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix}$$

Now,

$$A(\text{adj}A) = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}\begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

Illy

$$(\text{adj}A)A = \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

$$|A|I = 11\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

Hence $A(\text{adj}A) = (\text{adj}A)A = |A|I$

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23/3/20 ML Aggarwal **HW**

1) If $A = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$ & $\text{adj}A = A'$, then find the Value of $5a+b$ [5]

2) find the adjoint of $A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ and hence Show that $A(\text{adj}A) = |A| I_3$

3) If $A = \begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix}$, find $A(\text{adj}A)$

4) $A = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$ and $A(\text{adj}A) = K\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ find the Value of K.

5) If $A = \begin{bmatrix} 2 & -1 & 3 \\ 4 & 2 & 5 \\ 0 & 4 & -1 \end{bmatrix}$, verify $A(\text{adj}A) = 0$

6) If $A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$, $a \neq 0$, find $|A \,\text{adj}A|$.

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