25/3/2020

MATRICES & DETERMINANTS

1) If $0 < x < \pi$ & matrix $\begin{bmatrix} 2\sin x & 3 \\ 1 & 2\sin x \end{bmatrix}$ is singular, find the values of x.

Sol: $\begin{bmatrix} 2\sin x & 3 \\ 1 & 2\sin x \end{bmatrix}$ is singular

$\Rightarrow 4\sin^2 x - 3 = 0$

$\Rightarrow \sin x = \pm \frac{\sqrt{3}}{2}$

$\Rightarrow \sin x = \frac{\sqrt{3}}{2}$ (As $0 < x < \pi$)

$\Rightarrow \sin x = \sin \pi/3$ or $\sin(\pi - \pi/3)$

$$\boxed{x = \pi/3 \ \& \ 2\pi/3}$$

2) If A is a square matrix of order 3: $|A| = 5$ find $|2A|$ & $|3A'|$

Sol As we know $|KA| = K^n|A|$

$\therefore |2A| = 2^3|A| = 8 \times 5 = 40$

& $|3A'| = 3^3|A'| = 27 \times 5 = 135$ $[|A'| = |A|]$

3) If A is a square matrix of order 3 & $|A| = 7$, find $|adj A|$ & $|2\, adj A|$

Sol $|adj A| = |A|^{n-1} = (7)^{3-1} = (7)^2 = 49$

& $|2\, adj A| = 2^n\, |adj A| = 2^3 |A|^{n-1}$
$= 8 \times 49 = 392$

4) If A is a square matrix of order 3: $|adj A| = 225$ find $|A'|$

Sol $|adj A| = |A|^{n-1}$ $\Rightarrow |A| = \pm 15$

$225 = |A|^2$ $\Rightarrow |A'| = \pm 15$

5) If the matrix $A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 1 & -2 \\ 1 & 0 & 3 \end{bmatrix}$, find $|adj A|$ without Calculating $adj A$

Sol: order of matrix is 3 & we know

$|adj A| = |A|^{n-1} = |A|^{3-1} = |A|^2$ —— ①

Now

$$|A| = 1\begin{vmatrix} 1 & -2 \\ 0 & 3 \end{vmatrix} + 1\begin{vmatrix} 3 & -2 \\ 1 & 3 \end{vmatrix} + 2\begin{vmatrix} 3 & 1 \\ 1 & 0 \end{vmatrix}$$

$$= 1(3-0) + 1(9+2) + 2(0-1)$$

$$= 3 + 11 - 2 = 12 \neq 0$$

by ①,

$$|adj A| = (12)^2 = 144$$

6) If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, find $|A\, adj A|$.

Sol $|A| = 4 - 6 = -2 \neq 0$

$|A\, adj A| = |A|\,|adj A| = |A|\,|A|^{2-1} = |A|^2$

$\therefore |A\, adj A| = (-2)^2 = 4.$

7) If $P = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$ is the adjoint of a 3×3 matrix & $|A| = 4$ find α.

Sol:

$|adj A| = |A|^{n-1} = |A|^2 = (4)^2 = 16$

$\Rightarrow |P| = 16$

$\Rightarrow 1\begin{vmatrix} 3 & 3 \\ 4 & 4 \end{vmatrix} - \alpha\begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix} + 3\begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix} = 16$

$\Rightarrow 0 - \alpha(-2) + 3(-2) = 16$

$2\alpha - 6 = 16$

$2\alpha = 22$

$\boxed{\alpha = 11}$

8) If $A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$, Show that $A'A^{-1} = \begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$

Sol: $|A| = 1 + \tan^2 x \neq 0$

$\text{adj} A = \begin{bmatrix} 1 & -\tan x \\ -\tan x & 1 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} \text{adj} A = \frac{1}{1+\tan^2 x} \begin{bmatrix} 1 & -\tan x \\ +\tan x & 1 \end{bmatrix}$

Now,

$A'A^{-1} = \begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix} \left(\frac{1}{1+\tan^2 x} \begin{bmatrix} 1 & -\tan x \\ +\tan x & 1 \end{bmatrix} \right)$

$= \frac{1}{1+\tan^2 x} \begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix} \begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix}$

$= \frac{1}{1+\tan^2 x} \begin{bmatrix} 1-\tan^2 x & -2\tan x \\ 2\tan x & 1-\tan^2 x \end{bmatrix}$

$= \begin{bmatrix} \frac{1-\tan^2 x}{1+\tan^2 x} & \frac{-2\tan x}{1+\tan^2 x} \\ \frac{2\tan x}{1+\tan^2 x} & \frac{1-\tan^2 x}{1+\tan^2 x} \end{bmatrix}$

$= \begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$

9) Find the matrix X: $\begin{bmatrix} 1 & -4 \\ 3 & -2 \end{bmatrix} X = \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$

Sol: Let $A = \begin{bmatrix} 1 & -4 \\ 3 & -2 \end{bmatrix}$ & $B = \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$

we get, $AX = B$.

$A^{-1}AX = A^{-1}B$ [Pre multiply by A^{-1}]

$IX = A^{-1}B$ $[A^{-1}A = I]$

$\begin{bmatrix} 6 & 2 \\ \frac{1}{2} & \end{bmatrix}$ $\boxed{X = A^{-1}B} \longrightarrow$ (Do Yourself)

10) find the matrix A :

Ans
$$\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Let $B = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$ & $C = \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}$

we get

$$BAC = I.$$

$$B^{-1}BAC = B^{-1}I \quad [\text{Pre multiply by } B^{-1}]$$

$$IAC = B^{-1}$$

$$AC = B^{-1}$$

$$ACC^{-1} = B^{-1}C^{-1} \quad [\text{Post multiply by } C^{-1}]$$

$$AI = B^{-1}C^{-1} \quad \text{—— (1)}$$

$$A = B^{-1}C^{-1}$$

Now $|B| = 4-3 = 1$ & $|C| = 9-10 = -1$

adj $B = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}$ & adj $C = \begin{bmatrix} -3 & -2 \\ -5 & -3 \end{bmatrix}$

$B^{-1} = 1\begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}$ & $C^{-1} = \frac{-1}{1}\begin{bmatrix} -3 & -2 \\ -5 & -3 \end{bmatrix}$

$C^{-1} = \begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix}$

from (1)

$$A = B^{-1}C^{-1} = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

HW (25/3/20)

1) Find a 2x2 matrix B : $B\begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$

2) Find matrix P: $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} P \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$

3) Show that the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$ Satisfies $A^3 - 6A^2 + 5A + 11I = 0$

Hence find A^{-1}.

4) Find A^{-1} if $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$. Also show that $A^{-1} = \frac{A^2 - 3I}{2}$

5) $A = \begin{bmatrix} 5 & 0 & 4 \\ 2 & 3 & 2 \\ 1 & 2 & 1 \end{bmatrix}$, $B^{-1} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ find $(AB)^{-1}$.